PHYSICAL REVIEW E **73**, 010302(R) (2006)

Collision frequencies and energy flux in a dilute granular gas

Sébastien Aumaître and Stéphan Fauve[1,*]
[1]*Laboratoire de Physique Statistique de l'Ecole Normale Supérieure, CNRS UMR 8550, 24 Rue Lhomond, 75231 Paris Cedex 05, France*

(Received 25 October 2005; revised manuscript received 19 December 2005; published 27 January 2006)

Recent experimental study of a granular gas fluidized by vibrations in a low gravity environment has reported that the collision frequency ν_P of the particles with the container boundary scales roughly like N^α with $\alpha=0.6\pm0.1$, where N is the number of particles. Using numerical simulations, we show that this scaling is observed on a wide range of N, both for ν_P and for the particle-particle collision frequency ν_c. Simple scaling arguments show that this behavior is related to the energy flux in the granular gas, from injection by the moving boundary to dissipation by inelastic collisions. We predict in the dilute limit that the collision frequencies scale such as $\sqrt{N}$ are in fair agreement with experimental measurements.

DOI: 10.1103/PhysRevE.73.010302 PACS number(s): 45.70.−n, 81.70.Ha, 45.50.−j, 83.10.Pp

A granular gas, i.e., a collection of macroscopic particles fluidized by vibration, is one of the elementary examples of granular flows. Each collision between particles being inelastic, a continuous input of energy is required to reach an out of equilibrium stationary regime. This is usually achieved by vibrating the bottom boundary of the container. Several experiments have been performed to study the dependence of the granular temperature (i.e., the kinetic energy per particle) or the pressure on the vibration parameters, the velocity statistics of the particles, the density profile of the gas, and the formation of clusters of particles [1]. A type of measurement has been recently performed. Instead of looking at bulk properties, the collision frequency ν_P of the particles with the container boundary has been recorded for a dilute granular gas fluidized by vibrations in a low gravity environment [2]. It has been found that $\nu_P \propto VN^\alpha$ with $\alpha=0.6\pm0.1$, where $V=A\omega$ is the vibration velocity (of amplitude A and angular frequency ω). The linear dependence on V results from the existence of only one time scale in a low gravity experiment, the period of vibration $2\pi/\omega$. The dependence on N is more puzzling. It strongly differs from the case of a molecular gas, for which it is linear. We show here that this is confirmed by numerical simulations which give $\alpha\approx0.5$ on a wider range of N than the experimentally studied one. In addition, the particle-particle collision frequency per particle, ν_c, displays a similar scaling law. This traces back to the energy flux budget in the granular gas, from injected power by the moving boundary to dissipated one by inelastic collisions. We thus show that this scaling law is related to the dissipative nature of collisions and understand it with a simple model in the limit of a low density granular gas.

A two-dimensional (2D) granular gas is simulated with an event driven molecular dynamics method [3]. N disks of mass m and diameter d are enclosed in a square box of size L. Energy input is provided by one vibrating wall with a symmetric sawtooth motion, $y=\pm Vt$, of period $T=2\pi/\omega$. Particle rotation is neglected in our simulations. We take m, d, and T, respectively, as unit of mass, length, and time. In these units, we take $V=6$. As in most previous studies, particle-wall collisions are elastic whereas inelastic binary collisions between particles are modeled with a constant restitution coefficient r, where r is the ratio between precollisional and postcollisional normal relative velocities. We take $L=100d$ and increase the particle number from $N=2$ to $N=600$. Thus, we have for the mean particle density, $n_0=N/L^2$, $0.0004<n_0<0.02$, for the number of particle layers at rest, $n_c=Nd/L$, $0.02<n_c<6$, and for the mean free path $l_0=1/(2\sqrt{2}n_0d)$ corresponding to n_0, $0.06L<l_0<18L$.

As in experiments [2], we record the collision frequency ν_P of the particles with the boundary ($y=L$), opposite to the vibrating one. We also record the collision frequency ν_c of one particle with the others for $r=0.95$, $r=0.9$, and $r=0.8$. The first value corresponds to the experiments performed with steel spheres [2]. The numerical results for the collision frequencies ν_P and ν_c for the different values of r can be collapsed on single curves for N not too large if we plot $\nu_P\sqrt{1-r^2}$ and $\nu_c\sqrt{1-r^2}$ versus N. This is displayed in Fig. 1. We observe that $\nu_P \propto \sqrt{N}$ in agreement with experimental

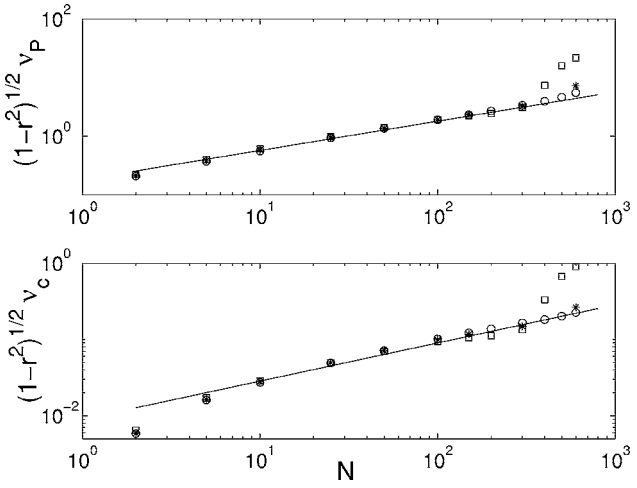

FIG. 1. Collision frequency, ν_P, of the particles with the upper boundary, times $\sqrt{1-r^2}$ with $r=0.95$ (○), $r=0.9$ (*), and $r=0.8$ (□) as a function of the number of particles N. Collision frequency, ν_c, of a particle with other particles, times $\sqrt{1-r^2}$ with $r=0.95$ (○), $r=0.9$ (*), and $r=0.8$ (□). Full lines with slope 1/2.

*Corresponding author. Email address: fauve@lps.ens.fr

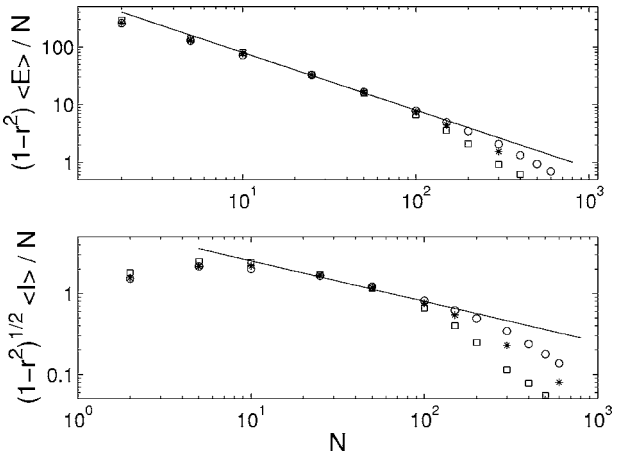

FIG. 2. Granular temperature $\langle E\rangle/N$ as a function of N for $r=0.95$ (○), $r=0.9$ (*), and $r=0.8$ (□). Full line of slope −1. Mean injected power per particle, $\langle I\rangle/N$, as a function of N for $r=0.95$ (○), $r=0.9$ (*), and $r=0.8$ (□). Full line of slope −1/2.

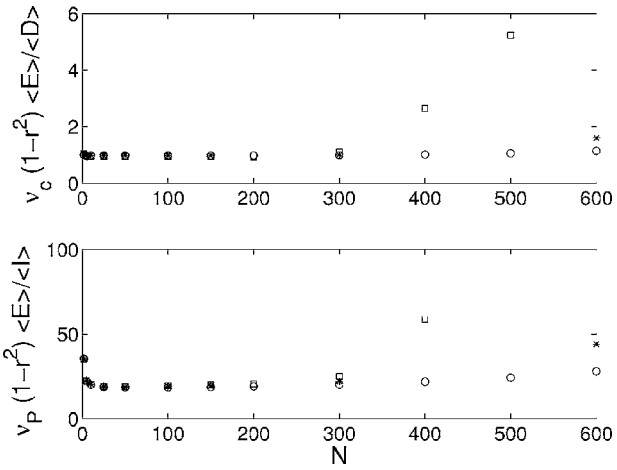

FIG. 3. Normalized collision frequency, $\nu_c(1-r^2)\langle E\rangle/\langle D\rangle$ as a function of N for $r=0.95$ (○), $r=0.9$ (*), and $r=0.8$ (□). Normalized collition frequency, $\nu_P(1-r^2)\langle E\rangle/\langle I\rangle$ as a function of N for $r=0.95$ (○), $r=0.9$ (*), and $r=0.8$ (□).

measurements and similarly that $\nu_c \propto \sqrt{N}$, on nearly two decades for $r=0.95$ and $r=0.9$. The lower limit of this behavior is roughly $N\approx 10$ particles. Velocities become larger and larger (and anisotropic) in the limit of small N. Thus, despite the small probability of collision between particles, the gas dissipates enough power to reach a statistically stationary regime. The mean density profile along the y axis, perpendicular to the vibrating boundary, stays roughly constant up to $N\approx 100$ particles with the realistic values of r that are considered. When N is increased further, the y dependence of the mean density cannot be neglected anymore, such that the mean free path l between binary collisions can be large compared to the diameter d of particles close to the moving boundary whereas it becomes smaller than d far from away [4]. This strong inhomogeneity is characteristic of granular gas and results from energy dissipation. The granular medium becomes cooler and thus denser, far from the moving boundary which provides energy [5]. The resulting inhomogeneity increases when the volume is increased at constant density, thus in the thermodynamic limit [4]. The upper limit of the scaling behavior we observe roughly corresponds to the clustering instability leading to a nonhomogeneous density profile also along the direction perpendicular to the energy flux, i.e., the x axis [6]. This occurs for a smaller value of N when r is decreased. The limit of the range of validity of the scaling for large N is clearly observed for $r=0.8$ in Fig. 1. The collision frequencies abruptly increase as soon as a cluster develops close to one of the upper corners of the container. However, we emphasize that it is remarkable to observe a scaling behavior even when the density of the granular gas becomes strongly nonhomogeneous along the axis parallel to the energy flux as long as no clustering instability occurs.

The mean power $\langle I\rangle$ injected by the moving boundary into the granular gas is related to the collision frequency with the boundary ν_P and the granular temperature $\langle E\rangle/N$ whereas the mean dissipated power $\langle D\rangle$ is related to ν_c and $\langle E\rangle/N$. $\langle E\rangle/N$ and $\langle I\rangle/N$ are shown in Fig. 2. We observe that they do not display any scaling law with respect to N on a range comparable to the one of Fig. 1.

$\langle E\rangle$ and $\langle I\rangle=\langle D\rangle$ both increase with N for N small and then decrease for N large. They reach their maximum for different values of N as can be understood as follows. The relation between $\langle D\rangle$, $\langle E\rangle$, and ν_c can be obtained by observing that an amount of energy $(1-r^2)\langle E\rangle/N$ is on average lost at each binary collision. Thus, one expects for the mean dissipated power,

$$\langle D\rangle \propto (1-r^2)\langle E\rangle\nu_c. \tag{1}$$

Since $\langle E\rangle$, $\langle D\rangle$, and ν_c all depend on N and ν_c increases with N, it follows from (1) that $\langle E\rangle$ reaches its maximum before $\langle I\rangle$ or $\langle D\rangle$ reach their maximum when N is increased.

We emphasize that fluctuations are neglected in (1), so the accuracy of this estimate should be checked. Figure 3 shows that Eq. (1) is fairly accurate when N is not too large, the constant of proportionality being close to 1. We also observe that a similar relation holds for the collision frequency ν_P with the boundary, which we will comment on below. As in Fig. 1, we observe that the upper limit of validity of the scaling law decreases to smaller N when r is decreased.

In order to go further, we need to evaluate the injected power $\langle I\rangle$. This has been addressed in several papers, with or without gravity [7–10]. It is a difficult problem, first because it requires the knowledge of the probability density function (PDF) of the velocity of the particles, which do not display a universal shape for granular gas, in particular close to the moving boundary. Second, even if an approximate PDF is chosen, one should also know how the colliding particle velocity and the motion of the boundary are correlated (or not). A way to eliminate this problem is to consider an asymmetric sawtooth driving with ascending motion of the piston at velocity V and infinitely rapid descending motion. Thus, all particles collide with the boundary with velocity $+V$, and it has been shown that the scaling $\langle I\rangle \propto pLV$ where p is the pressure at the boundary is very accurate [8]. Then, assuming $p\propto\langle E\rangle/L^2$, gives $\langle I\rangle\propto V\langle E\rangle/L$. It is rather difficult to achieve such a motion experimentally and a more realistic approxi-

mation to the roughly sinusoidal driving used most of the time is a symmetric sawtooth forcing. The above scaling or a slightly modified one can be still valid if the root mean square particle velocity v_{rms} is small compared to V. However, for a dilute granular gas with $r \approx 1$, we can have $v_{rms} \gg V$ [11], such that the probabilities of collision with the piston at velocities $\pm V$ become nearly equal. Then, the coefficient of the linear term in V vanishes in the mean injected power $\langle I \rangle$ [7,9]. This remains true for all symmetrical excitations including sinusoidal vibration. We get $\langle I \rangle \propto mV^2 \nu_P$, with $\nu_P \propto n(0) L v_{rms}$ where $n(0)$ is the density of particles close to the moving boundary. Thus, if we assume that the density is roughly homogeneous in the dilute limit $[n(0) \approx n_0]$, we get

$$\langle I \rangle \approx C_1 m V^2 n_0 L v_{rms}, \tag{2}$$

where C_1 is a numerical constant. We also have in this limit $\nu_c \approx C_2 n_0 d v_{rms}$, where C_2 is another numerical constant. For an homogeneous isotropic and Gaussian velocity distribution, we have $C_1 = 1/4$ and $C_2 = 2\sqrt{2}$. We do not expect quantitative agreement with these values in our numerical simulations. Even if nonhomogeneity can be taken into account to some extent by replacing n_0 by a local density $n(y)$, deviations from isotropy and from a Gaussian distribution are expected for a granular gas and modify the numerical values of C_1 and C_2.

Then, using (1) and (2), and $n_0 = N/L^2$, $v_{rms} = \sqrt{2\langle E \rangle / mN}$, we get

$$\langle E \rangle \approx \frac{C_1}{C_2} \frac{mV^2}{1-r^2} \frac{L}{d}, \tag{3}$$

$$\langle I \rangle = \langle D \rangle \approx C_1 \sqrt{\frac{2C_1}{C_2}} \frac{mV^3}{\sqrt{1-r^2}} \frac{\sqrt{N}}{\sqrt{Ld}}, \tag{4}$$

and

$$\nu_c \approx \frac{C_2}{C_1} \frac{d}{L} \nu_P \approx \sqrt{2C_1 C_2} \sqrt{\frac{d}{L}} \frac{V}{L\sqrt{1-r^2}} \sqrt{N}. \tag{5}$$

The scaling laws displayed in Fig. 1, $\sqrt{1-r^2}\nu_P \propto \sqrt{N}$ and $\sqrt{1-r^2}\nu_c \propto \sqrt{N}$, are thus explained with our simple model. The scaling with respect to the particle number results form the dissipative nature of the collisions, as emphasized by the following argument: taking into account only quantities varying on N, all the other parameters being fixed, we have $\nu_P \propto \nu_c \propto N v_{rms} \propto \sqrt{\langle E \rangle N}$, in molecular as well as in granular gases. In molecular gases, $\langle E \rangle \propto N$, thus we get the classical result $\nu_P \propto \nu_c \propto N$. In granular gases, $\langle E \rangle$ is determined by the energy budget, $\langle E \rangle \propto \langle I \rangle / (1-r^2)\nu_c \propto \nu_P / (1-r^2)\nu_c = \text{const}$. Thus, we get $\nu_P \propto \nu_c \propto \sqrt{N}$. This emphasizes that a granular gas driven by a vibrating boundary is very different from a molecular gas in contact with a heat bath.

The model also predicts that the granular temperature $\langle E \rangle / N$ scales like $1/N$ whereas the flux of energy per particle $\langle I \rangle / N = \langle D \rangle / N$ scales like $1/\sqrt{N}$, in reasonable agreement with Fig. 2 in an intermediate range of N. Their dependence on the restitution coefficient r is also well described since numerical data obtained for $r=0.95$, $r=0.9$, and $r=0.8$ collapse on a single curve provided that $(1-r^2)\langle E \rangle$ [respectively, $\sqrt{(1-r^2)}\langle I \rangle$] is plotted (see Fig. 2). However, the data obtained for $r=0.8$ show that the range of N for which this collapse is observed becomes smaller when r is decreased. Finally, the orders of magnitude of $\langle E \rangle$ and $\langle I \rangle$ are correctly estimated. If one compares $\nu_P / \nu_c = C_1 L / C_2 d$ with simulations which give a ratio roughly equal to 20 in the range where the scaling law for the collisions frequencies is observed (see Fig. 3), we get $C_2 \approx 5 C_1$. This gives for $r=0.95$, $\langle E \rangle \approx 7000$ instead of numerically computed values of order 8000 in the intermediate range of N.

This simple model has obvious limitations both for small and large N. The velocity distribution becomes strongly anisotropic for small N (ten particles or less). Not surprisingly, the component along the vibration axis has tails involving larger velocities than the perpendicular component. This explains why $\langle E \rangle$ decreases by 25% when N is decreased from 25 to five particles, instead of staying constant as predicted by the model. On the other side, the particle density becomes nonhomogeneous along the vibration axis when N becomes too large (typically $N > 100$, i.e., more than one layer of particles at rest, for the values of r used in our study). We emphasize that the validity of our model relies on the assumption of homogeneous density and velocity distribution and that deviations from it at large N or small r are due to the failure of this assumption. We also stress that the granular gas is never in a hydrodynamic regime within the parameter range of our study. For small N (say $N=25$) the density is homogeneous within 10% but the mean free path is comparable to the system size. For large N (say $N=300$), the density is nonhomogeneous and the mean free path varies by a factor 10 (roughly from $0.5L$ to $0.05L$) from the vicinity of the piston to the opposite boundary.

Finally, instead of changing N at constant volume, as done above, it is instructive to consider the behavior of the granular temperature $\langle E \rangle / N$ when N is increased at constant mean density n_0. Eq. (3) shows that

$$\frac{\langle E \rangle}{\sqrt{N}} = \frac{C_1}{C_2} \frac{mV^2}{1-r^2} \frac{1}{\sqrt{n_0 d}}, \tag{6}$$

thus $\langle E \rangle$ is not extensive. The granular gas behaves as if the effective number of degrees of freedom were $N_f \propto \sqrt{N}$ in fair agreement with the numerical simulations of Ref. [4]. A similar qualitative behavior has been recently found using a hydrodynamical description [12]. However, this approximation is not valid in our simulations, as already mentioned above.

Despite its limitations, it is remarkable that our simple model captures the correct scaling law for collision frequencies, observed both in numerical simulations and in experiments, and also describes correctly energy flux in the dilute regime.

We acknowledge A. Alastuey, F. Cornu, E. Falcon, A. Puglisi, F. van Wijland, and P. Visco for stimulating discussions and the Centre National d'Études Spatiales for support.

[1] S. Warr, J. M. Huntley, and G. T. H. Jacques, Phys. Rev. E **52**, 5583 (1995); A. Kudrolli, M. Wolpert, and J. P. Gollub, Phys. Rev. Lett. **78**, 1383 (1997); E. Falcon, S. Fauve, and C. Laroche, Eur. Phys. J. B **9**, 183 (1999); E. Falcon *et al.*, Phys. Rev. Lett. **83**, 440 (1999); J. S. Olafsen and J. S. Urbach, Phys. Rev. E **60**, R2468 (1999); F. Rouyer and N. Menon, Phys. Rev. Lett. **85**, 3676 (2000); A. Kudrolli and J. Henry, Phys. Rev. E **62**, R1489 (2000); R. D. Wildman, J. M. Huntley, and D. J. Parker, *ibid.* **63**, 061311 (2001); C. Huan, X. Yang, D. Candela, R. W. Mair, and R. L. Walsworth, *ibid.* **69**, 041302 (2004); J. S. van Zon *et al.*, *ibid.* **70**, 040301 (2004).

[2] E. Falcon *et al.*, *Collision Statistics in a Dilute Granular Gas Fluidized by Vibrations in Low Gravity*, submitted to Europhys. Lett. (unpublished).

[3] For a discussion about the validity of event driven simulations of a two-dimensional granular gas excited by vibrations, see S. Luding, H. J. Herrmann, and A. Blumen, Phys. Rev. E **50**, 3100 (1994) and references therein.

[4] S. Aumaître, J. Farago, S. Fauve, S. McNamara, Eur. Phys. J. B **42**, 255 (2004).

[5] The development of such nonhomogeneous density profiles in vibrated granular media has been emphasized many times starting from early hydrodynamic descriptions, P. K. Haff, J. Fluid Mech. **134**, 401 (1983).

[6] M. Argentina, M. G. Clerc, and R. Soto, Phys. Rev. Lett. **89**, 044301 (2002); E. Livne, B. Meerson, and P. V. Sasorov, Phys. Rev. E **65**, 021302 (2002); J. J. Brey, M. J. Ruiz-Montero, F. Moreno, and R. Garcia-Rojo, *ibid.* **65**, 061302 (2002).

[7] S. Warr and J. M. Huntley, Phys. Rev. E **52**, 5596 (1995).

[8] S. McNamara and J. L. Barrat, Phys. Rev. E **55**, 7767 (1997).

[9] V. Kumaran, Phys. Rev. E **57**, 5660 (1998).

[10] S. McNamara and S. Luding, Phys. Rev. E **58**, 813 (1998).

[11] For $r \geqslant 0.9$, we have always $v_{rms} \geqslant 2V$ even for $N=600$. For $r=0.8$, $v_{rms} \geqslant V/3$.

[12] P. Visco, A. Puglisi, A. Barrat, F. van Wijland, and E. Trizac, Eur. Phys. J. B (unpublished).